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Year: 2018

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## **Equilibrium in the symmetric two-player Hirshleifer contest: Uniqueness and characterization**

Ewerhart, Christian ; Sun, Guang-Zhen

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DOI: <https://doi.org/10.1016/j.econlet.2018.05.010>

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ZORA URL: <https://doi.org/10.5167/uzh-151546>

Journal Article

Accepted Version



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Originally published at:

Ewerhart, Christian; Sun, Guang-Zhen (2018). Equilibrium in the symmetric two-player Hirshleifer contest: Uniqueness and characterization. *Economics Letters*, 169:51-54.

DOI: <https://doi.org/10.1016/j.econlet.2018.05.010>

## Accepted Manuscript

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PII: S0165-1765(18)30183-6

DOI: <https://doi.org/10.1016/j.econlet.2018.05.010>

Reference: ECOLET 8045

To appear in: *Economics Letters*

Received date: 26 April 2018

Revised date: 9 May 2018

Accepted date: 10 May 2018

Please cite this article as: Ewerhart C., Sun G.-Z., Equilibrium in the symmetric two-player Hirshleifer contest: Uniqueness and characterization. *Economics Letters* (2018), <https://doi.org/10.1016/j.econlet.2018.05.010>

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**Highlights**

- The symmetric two-player Hirshleifer contest admits a unique equilibrium.
- The support of the equilibrium strategy is finite, and includes the origin.
- We establish a lower bound for the cardinality of the support.
- The undissipated rent approaches zero as the parameter grows to infinity.
- We also discuss ex-post overdissipation and extensions.

**Equilibrium in the Symmetric Two-Player  
Hirshleifer Contest: Uniqueness and Characterization**

Christian Ewerhart<sup>†</sup>

Guang-Zhen Sun<sup>‡</sup>

May 9, 2018

**Abstract.** The symmetric two-player Hirshleifer contest admits a unique equilibrium. The equilibrium support is finite and includes the zero expenditure level. We also establish a lower bound for the cardinality of the support and an upper bound for the undissipated rent.

**Keywords.** Contests · Mixed-strategy equilibrium · Rent dissipation · Uniqueness

**JEL codes.** C72, D72, D74

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## 1. Introduction

Mixed equilibria in contests of the generalized Tullock form, for which winning probabilities depend on the *ratio* of resources expended, have recently received much attention from theorists (Baye et al., 1994; Alcade and Dahm, 2010; Ewerhart, 2015, 2017a, 2017b; Feng and Lu, 2017). There is another appealing class of contests, however, where the winning probabilities depend instead on the *difference* of resources expended (Hirshleifer, 1989; Skaperdas, 1996; Baik, 1998; Che and Gale, 2000). In particular, Hirshleifer's framework has its merits for the analysis of military combat (Dupuy, 1987; Hirshleifer, 2000). Notwithstanding, the nature of mixed equilibria in that model has remained poorly understood.

In this paper, we prove uniqueness of the equilibrium in the symmetric two-player Hirshleifer contest, and offer a characterization of the mixed equilibrium. It is shown that the support of the symmetric equilibrium strategy is finite and includes the origin. Moreover, the cardinality of the support grows over any finite bound as the decisiveness parameter goes to infinity. Further, we show that the undissipated rent converges to zero as the decisiveness parameter goes to infinity, and that ex-post overdissipation may occur. We conclude by extending the uniqueness result to a larger class of contests.

The uniqueness result is stated in Section 2, and proven in Section 3. Section 4 characterizes the equilibrium. Rent dissipation is dealt with in Section 5. Section 6 discusses ex-post overdissipation. Alternative contest technologies are considered in Section 7.

## 2. Statement of the uniqueness result

The Hirshleifer contest is specified as follows. Each of two players  $i \in \{1, 2\}$  expends resources  $x_i \geq 0$  in an attempt to win a prize of normalized value one. Player  $i$ 's payoff is given as

$$\Pi_i(x_i, x_j) = \frac{\exp(\alpha x_i)}{\exp(\alpha x_i) + \exp(\alpha x_j)} - x_i \quad (1)$$

$$= \frac{1}{1 + \exp(\alpha(x_j - x_i))} - x_i, \quad (2)$$

where  $j \in \{1, 2\}$  with  $j \neq i$ , and  $\alpha > 0$  measures the decisiveness of the difference-form contest. In particular, for  $\alpha \rightarrow \infty$ , payoffs converge to those of the all-pay auction.

Any bid exceeding one is strictly dominated. We therefore define a *mixed strategy* for player  $i$  as a probability measure  $\mu_i$  on the Borel subsets of  $[0, 1]$ . The set of mixed strategies for player  $i$  will be denoted by  $M$ , where pure strategies  $x_i \in [0, 1]$  are interpreted as Dirac measures, as usual. Each player  $i$ 's expected payoff is well-defined for any  $(\mu_i, \mu_j) \in M \times M$ , and will, with some abuse of notation, be denoted by  $\Pi_i(\mu_i, \mu_j)$ . An *equilibrium* is a pair  $\mu^* = (\mu_1^*, \mu_2^*) \in M \times M$  such that  $\Pi_i(\mu_i^*, \mu_j^*) \geq \Pi_i(\mu_i, \mu_j^*)$  for any  $i, j \in \{1, 2\}$  with  $j \neq i$ , and for any  $\mu_i \in M$ .

**Proposition 1.** *For any  $\alpha > 0$ , the Hirshleifer contest with parameter  $\alpha$  has a unique equilibrium.*

## 3. Proof of Proposition 1

Equilibrium existence is known (cf. Hirshleifer, 1989, fn. 12). The proof of uniqueness starts from the following observation.

**Lemma 1.** *Let  $\mu = (\mu_1, \mu_2) \in M \times M$ . Then, for any  $i, j \in \{1, 2\}$  with  $j \neq i$ , the set of maximizers  $X_i(\mu) = \arg \max_{\tilde{x}_i \in [0, 1]} \Pi_i(\tilde{x}_i, \mu_j)$  is finite.*

**Proof.** The proof is a straightforward adaption of Ewerhart (2015, Th. 3.2), and therefore omitted.<sup>1</sup>  $\square$

Next, we show the following.

**Lemma 2.** *The set  $X^\alpha = \bigcap_{\mu^* \text{ equilibrium}} X_1(\mu^*)$  is nonempty, and contains the support of any equilibrium strategy (for both players).*

**Proof.** Take an equilibrium  $\mu^* = (\mu_1^*, \mu_2^*)$ . Clearly, the support of  $\mu_1^*$  is a subset of  $X_1(\mu^*)$ . Let  $\mu^{**} = (\mu_1^{**}, \mu_2^{**})$  be an arbitrary equilibrium. Then, since equilibria in two-player contests are interchangeable (Ewerhart, 2017b, Appendix),  $(\mu_1^*, \mu_2^{**})$  is an equilibrium. Therefore, the support of  $\mu_1^*$  is a subset of  $X_1(\mu_1^*, \mu_2^{**})$ . But  $X_1(\mu_1^*, \mu_2^{**}) = X_1(\mu^{**})$ . Hence, the support of  $\mu_1^*$  is contained in  $X_1(\mu^{**})$  for any equilibrium  $\mu^{**}$ . In particular,  $X^\alpha \neq \emptyset$ . The second claim follows by symmetry.  $\square$

Denote by  $K = |X^\alpha|$  the number of elements of  $X^\alpha$ . Thus,  $X^\alpha = \{z_1, \dots, z_K\}$ , where  $z_1 > z_2 > \dots > z_K$ . Suppose first that  $K = 1$ . Then, the equilibrium is obviously unique. Suppose next that  $K \geq 2$ . Fix some equilibrium  $\mu^* = (\mu_1^*, \mu_2^*)$ , and let  $p_j^m = \mu_i^*(\{z_m\}) \geq 0$  denote the weight assigned by  $\mu_j^*$  to  $z_m$ , for  $j \in \{1, 2\}$  and  $m \in \{1, \dots, K\}$ . We know that  $z_1, \dots, z_K$  all deliver the equilibrium payoff  $\Pi_i^*$  against  $\mu_j^*$ , i.e.,

$$\Pi_i^* = \left( \sum_{m=1}^K p_j^m \frac{\exp(\alpha z_k)}{\exp(\alpha z_k) + \exp(\alpha z_m)} \right) - z_k \quad (k = 1, \dots, K; j \neq i). \quad (3)$$

Thus, there are  $K$  equations to identify  $(K + 1)$  unknowns  $p_j^1, \dots, p_j^K$  and  $\Pi_i^*$ . Notably, adding the relationship  $\sum_{m=1}^K p_j^m = 1$  does not help in general. Instead, we focus on the *largest* element of the support of player  $i$ 's equilibrium strategy.<sup>2</sup> Since  $K \geq 2$ , we know that  $z_1$  is an interior maximum.

<sup>1</sup>If attention is restricted to strategies that are absolutely continuous with respect to the Lebesgue measure, the use of complex-analytic methods may be circumvented (Sun, 2017).

<sup>2</sup>The first-named author would like to thank Larry Samuelson for this suggestion.

Hence, the first-order condition implies

$$\sum_{m=1}^K p_j^m \frac{\alpha \exp(\alpha z_1) \exp(\alpha z_m)}{(\exp(\alpha z_1) + \exp(\alpha z_m))^2} = 1. \quad (4)$$

Combining these  $(K + 1)$  equations yields

$$\begin{pmatrix} \frac{\exp(\alpha z_1)}{\exp(\alpha z_1) + \exp(\alpha z_1)} & \cdots & \frac{\exp(\alpha z_1)}{\exp(\alpha z_1) + \exp(\alpha z_K)} & 1 \\ \vdots & \ddots & \vdots & \vdots \\ \frac{\exp(\alpha z_K)}{\exp(\alpha z_K) + \exp(\alpha z_1)} & \cdots & \frac{\exp(\alpha z_K)}{\exp(\alpha z_K) + \exp(\alpha z_K)} & 1 \\ \frac{\alpha \exp(\alpha z_1) \exp(\alpha z_1)}{(\exp(\alpha z_1) + \exp(\alpha z_1))^2} & \cdots & \frac{\alpha \exp(\alpha z_1) \exp(\alpha z_K)}{(\exp(\alpha z_1) + \exp(\alpha z_K))^2} & 0 \end{pmatrix} \begin{pmatrix} p_j^1 \\ \vdots \\ p_j^K \\ -\Pi_i^* \end{pmatrix} = \begin{pmatrix} z_1 \\ \vdots \\ z_K \\ 1 \end{pmatrix}. \quad (5)$$

It turns out that (5) has at most one solution.

**Lemma 3.** *The square matrix on the left-hand side of (5) is invertible.*

**Proof.** Let  $e_k = \exp(\alpha z_k)$  for  $k = 1, \dots, K$ , and

$$A_1 = \begin{pmatrix} \frac{e_1}{e_1 + e_1} & \cdots & \frac{e_1}{e_1 + e_K} & 1 \\ \frac{e_2}{e_2 + e_1} & \cdots & \frac{e_2}{e_2 + e_K} & 1 \\ \vdots & \ddots & \vdots & \vdots \\ \frac{e_K}{e_K + e_1} & \cdots & \frac{e_K}{e_K + e_K} & 1 \\ \frac{\alpha e_1 e_1}{(e_1 + e_1)^2} & \cdots & \frac{\alpha e_1 e_K}{(e_1 + e_K)^2} & 0 \end{pmatrix}. \quad (6)$$

Subtracting row  $k = 1$  from row  $k$ , for  $k = 2, \dots, K$ , yields  $\det A_1 = \det A_2$ ,

where

$$A_2 = \begin{pmatrix} \frac{e_1}{e_1 + e_1} & \cdots & \frac{e_1}{e_1 + e_K} & 1 \\ \frac{(e_2 - e_1)e_1}{(e_2 + e_1)(e_1 + e_1)} & \cdots & \frac{(e_2 - e_1)e_K}{(e_2 + e_K)(e_1 + e_K)} & 0 \\ \vdots & \ddots & \vdots & \vdots \\ \frac{(e_K - e_1)e_1}{(e_K + e_1)(e_1 + e_1)} & \cdots & \frac{(e_K - e_1)e_K}{(e_K + e_K)(e_1 + e_K)} & 0 \\ \frac{\alpha e_1 e_1}{(e_1 + e_1)^2} & \cdots & \frac{\alpha e_1 e_K}{(e_1 + e_K)^2} & 0 \end{pmatrix}. \quad (7)$$

Next, we extract the factor  $e_m/(e_1 + e_m) > 0$  from column  $m$ , for  $m = 1, \dots, K$ , and the factor  $(e_k - e_1) > 0$  from row  $k$ , for  $k = 2, \dots, K$ . Further, we extract the factor  $\alpha e_1 > 0$  from the last row. This yields

$$\det A_2 = \left( \prod_{1 \leq m \leq K} \frac{e_m}{e_1 + e_m} \right) \cdot \left( \prod_{2 \leq k \leq K} (e_k - e_1) \right) \cdot \alpha e_1 \cdot \det A_3, \quad (8)$$



where

$$A_3 = \begin{pmatrix} \frac{e_1}{e_2+e_1} & \frac{e_1}{e_2+e_2} & \dots & \frac{e_1}{e_2+e_K} & 1 \\ \frac{e_1}{e_2+e_1} & \frac{e_2}{e_2+e_2} & \dots & \frac{e_K}{e_2+e_K} & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \frac{1}{e_K+e_1} & \frac{1}{e_K+e_2} & \dots & \frac{1}{e_K+e_K} & 0 \\ \frac{1}{e_1+e_1} & \frac{1}{e_1+e_2} & \dots & \frac{1}{e_1+e_K} & 0 \end{pmatrix}. \quad (9)$$

Finally, we exchange row 1 and row  $K+1$ . Therefore,  $\det A_3 = -\det A_4$ , where  $A_4 = \{\frac{1}{e_k+e_m}\}_{k=1,\dots,K;m=1,\dots,K}$  is a Cauchy matrix (e.g., Krattenthaler, 2001) with

$$\det A_4 = \frac{\prod_{1 \leq k < m \leq K} (e_k - e_m)^2}{\prod_{1 \leq k \leq K, 1 \leq m \leq K} (e_k + e_m)} \neq 0. \quad (10)$$

This proves the lemma.  $\square$

Recall that the support of *any* equilibrium is contained in  $X^\alpha = \{z_1, \dots, z_K\}$ . Hence, with probabilities  $p_j^1, \dots, p_j^K$  being unique, there can indeed be at most one equilibrium.

#### 4. Characterization<sup>3</sup>

Since the Hirshleifer contest with parameter  $\alpha > 0$  admits only one equilibrium, the two players necessarily use the same equilibrium strategy  $\mu_\alpha \in M$ . The following result characterizes  $\mu_\alpha$ .

**Proposition 2.** *Let  $\alpha > 0$ . Then, the following properties hold:*

- (i)  $\mu_\alpha$  has finite support  $\{y_1, \dots, y_L\}$ , where  $y_1 > \dots > y_L$ , with  $L \geq \frac{\alpha}{4}$ .
- (ii)  $\mu_\alpha$  has a mass point at the zero bid, i.e.,  $y_L = 0$ .
- (iii) there is two-sided peace<sup>4</sup> (i.e.,  $L = 1$ ) if and only if  $\alpha \leq 4$ .

<sup>3</sup>This section and the next supersede the corresponding parts of earlier work by the authors (Ewerhart, 2014; Sun, 2017).

<sup>4</sup>*Two-sided peace* (Hirshleifer, 1989) refers to the pure-strategy Nash equilibrium in which both contestants choose a zero resource commitment.

**Proof.** (i) By Lemma 1, the support of  $\mu_\alpha$  is finite. Denote by  $q_m = \mu_\alpha(\{y_m\}) > 0$  the probability assigned to  $y_m$ , for  $m \in \{1, \dots, L\}$ . From the KKT conditions,

$$\frac{q_l}{4} + \sum_{\substack{m=1 \\ m \neq l}}^L \frac{q_m \exp(\alpha y_m) \exp(\alpha y_l)}{(\exp(\alpha y_l) + \exp(\alpha y_m))^2} \leq \frac{1}{\alpha} \quad (l = 1, \dots, L), \quad (11)$$

with equality for  $l = 1, \dots, L-1$ , so that  $q_l \leq \frac{4}{\alpha}$  for any  $l = 1, \dots, L$ . Therefore,  $L \geq \frac{\alpha}{4}$ . (ii) For  $L = 1$ , the claim is due to Hirshleifer (1989). Suppose next that  $L \geq 2$ . Then, player  $i$ 's expected payoff against  $\mu_\alpha$  at the *smallest* mass point  $y_L$  satisfies

$$\begin{aligned} & \frac{\partial^2 \Pi_i(y_L, \mu_\alpha)}{\partial x_i^2} \\ &= \sum_{m=1}^{L-1} \frac{q_m \alpha^2 (\exp(\alpha y_m) - \exp(\alpha y_L)) \exp(\alpha y_L) \exp(\alpha y_m)}{(\exp(\alpha y_L) + \exp(\alpha y_m))^3} \end{aligned} \quad (12)$$

$$> 0, \quad (13)$$

which shows that  $y_L$  cannot be an interior maximum. Hence,  $y_L = 0$ . (iii) Hirshleifer (1989) has shown that  $x_1^* = x_2^* = 0$  is a pure-strategy Nash equilibrium for  $\alpha \leq 4$ . For  $\alpha > 4$ , however, part (i) implies  $L \geq 2$ .  $\square$

**Example ( $L = 2$ ).** Consider an equilibrium strategy  $\mu_\alpha$  that places probability  $q_1 > 0$  on  $y_1 > 0$ , and probability  $1 - q_1 > 0$  on  $y_2 = 0$ . Then,

$$y_1 = \frac{1}{2} - \frac{1}{1 + \exp(\alpha y_1)}, \quad (14)$$

$$q_1 = 1 - \frac{\alpha - 4}{4\alpha y_1^2}. \quad (15)$$

This equilibrium exists numerically for  $\alpha \in (4, 6.79)$ .<sup>5</sup>

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<sup>5</sup>The implicit value for  $y_1$  may be characterized alternatively in terms of the  $r$ -Lambert function (Mesö and Baricz, 2017).

## 5. Rent dissipation

Rent dissipation is always incomplete in the Hirshleifer contest.<sup>6</sup> For  $\alpha \rightarrow \infty$ , however, the equilibrium payoff  $\Pi^*$  goes to zero, as the following result shows.

**Proposition 3.**  $\Pi^* \leq \frac{2}{\alpha}$ .

**Proof.**

$$\Pi^* = \Pi_i(0, \mu_\alpha) \quad (16)$$

$$= \sum_{l=1}^L \frac{q_l}{1 + \exp(\alpha y_l)} \quad (17)$$

$$\leq 2 \sum_{l=1}^L \frac{q_l}{1 + \exp(\alpha y_l)} \cdot \frac{\exp(\alpha y_l)}{1 + \exp(\alpha y_l)} \quad (18)$$

$$= \frac{2}{\alpha} \left( \frac{\partial \Pi_i(0, \mu_\alpha)}{\partial x_i} + 1 \right). \quad (19)$$

Since  $\partial \Pi_i(0, \mu_\alpha) / \partial x_i \leq 0$ , the claim follows.  $\square$

Figure 1 outlines the equilibrium payoff  $\Pi^*$  and its upper bound as a function of  $\alpha$ . Note that  $\Pi^*$ , contrary to intuition, is not globally declining. For example, if  $\alpha = 6.1$  ( $< 6.6$ ), the equilibrium is given by  $y_1 = 0.4337$  (0.4517),  $q_1 = 0.5425$  (0.5173), and  $\Pi^* = 0.2646$  ( $< 0.2662$ ). Thus, the increase in  $y_1$  is more than compensated by a decline in  $q_1$ .

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<sup>6</sup>This fact contrasts, of course, with the complete rent dissipation pervasive in sufficiently decisive contests of the ratio form (Baye et al., 1994; Alcade and Dahm, 2010; Ewerhart, 2015, 2017a).

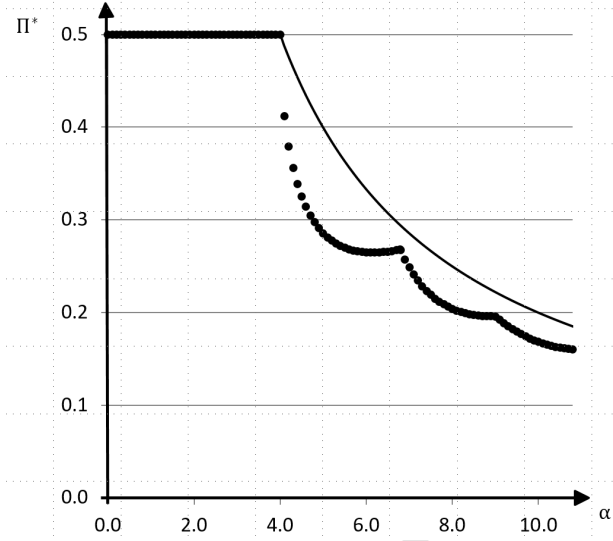


Figure 1. Undissipated rent as a function of  $\alpha$ .

## 6. Ex-post overdissipation

Although rent dissipation is less severe than in the Tullock case, the Hirshleifer contest may nevertheless feature ex-post overdissipation, i.e., the sum of realized bids may exceed the value of the prize with positive probability (cf. Baye et al., 1999).

**Proposition 4.**  $y_1 \geq \frac{3}{4} - \frac{10}{3\alpha}$ .

**Proof.** Recall that  $y_1 > \dots > y_L$ . Let  $L^*$  be the largest  $l \in \{1, \dots, L\}$  such that

$$\frac{\exp(\alpha y_1)}{\exp(\alpha y_1) + \exp(\alpha y_l)} < \frac{3}{4}, \quad (20)$$

and let

$$Q = \sum_{l=1}^{L^*} q_l. \quad (21)$$

Then, by Proposition 3,

$$\frac{2}{\alpha} \geq \Pi^* \geq \frac{3}{4}(1 - Q) + \frac{1}{2}Q - y_1. \quad (22)$$

Therefore,

$$Q \geq 3 - 4y_1 - \frac{8}{\alpha}. \quad (23)$$

But, from the first-order condition at  $y_1$ ,

$$1 \geq \alpha \sum_{l=1}^{L^*} q_l \frac{\exp(\alpha y_1) \exp(\alpha y_l)}{(\exp(\alpha y_1) + \exp(\alpha y_l))^2} \geq \alpha \cdot Q \cdot \frac{3}{4} \cdot \frac{1}{4}. \quad (24)$$

Using (23) in (24) yields the claim.  $\square$

Proposition 4 implies that ex-post overdissipation occurs for any sufficiently large  $\alpha$ . Using numerical analysis, we verified that  $y_1 > 0.5$  holds for  $\alpha > 7.2$ .

## 7. Alternative contest technologies

Consider a contest technology of the form

$$\Pi_i^h(x_i, x_j) = \frac{h(x_i)}{h(x_i) + h(x_j)} - x_i, \quad (25)$$

where  $h > 0$  is a positive impact function (as in Neary, 1997). Provided that  $h$  admits, in addition, a real-analytic extension to  $(-\varepsilon, \infty)$ , for some  $\varepsilon > 0$ , the uniqueness argument goes through. In that case, however, the equilibrium need no longer possess a mass point at the origin.<sup>7</sup>

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<sup>7</sup>E.g., there is no mass point at zero for  $h(x_i) = \eta + x_i$  if  $\eta < 1/4$  (Amegashie, 2006).

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